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Determining The Probability of Changing Grades from The Mathematical Statistics II Prerequisites Course Using The Markov Chain Process

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ABSTRACT

Courses are a place for study material that must be studied by students and must be delivered by a lecturer. Courses are an important element in the smallest unit of student learning transactions, which are served by educational institutions to measure their achievement index. The course achievement index is seen based on the value intervals set by the educational institution. Udayana University is one of the educational institutions in Indonesia and is famous in Bali for one of its study programs, namely the mathematics study program which is within the scope of the Faculty of Mathematics and Natural Sciences. The Udayana University Mathematics Study Program has 8 assessment letter scales, namely A, B+, B, C+, C, D+, D, E with the provision that the achievement index for each course is above the letter scale C. One of the purposes of having an achievement index is that it can be used in determine the taking of prerequisite courses for other courses.

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BACKGROUND

Prerequisite courses are courses that must be taken before taking the courses that require them. In this research, one course will be taken, namely Mathematical Statistics 2 as an advanced course with Basic Statistics and Mathematical Statistics 1 as prerequisite courses.

Prerequisite courses are courses that must be taken before taking courses that require them, where students can take one course more than once, which means that if a student gets a C or D grade, they are given the opportunity to improve their grade in that course or can Also immediately take courses that require it . In this case, students who initially get a C or D grade still have the possibility of getting an E grade

In this case, the author is interested in knowing the Probability of Changes in Values from the Prerequisite Course in Mathematical Statistics 2 with the Basic Statistics and Mathematical Statistics 1 courses as the prerequisite courses. This research will use the Markov Chain method to determine changes in prerequisite course grades for mathematical statistics II courses.

THEORETICAL BASIS

Markov Chain (Markov Chain)

Markov chains are a common mathematical technique used for modeling (modeling) various systems and business processes. This technique can be used to anticipate future changes in dynamic variables based on changes in dynamic variables in the past. This technology can also be used to analyze events including the future mathematically.

Markov Chain Theory was first discovered by Andrey Andreyevich Markov in 1906. He is a mathematician from Russia. He was a student of Chebysev, a person who was famous in the world of probability because of the formula he had discovered.

stochastic { $X_n$process } is said to have Markov properties if for $n = 0,1,2, \ldots$,that $X_n = i$reason the process is said to be in state $i$at time n and is a random variable defined as a sequence $i, j, i_0, i_1, \ldots i_{n+1}$ then it applies:

$$P\{X_{n+1} = j | X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}, X_n = i\} = P\{X_{n+1} = j | X_n = i\}$$

For all possible values of$i_0, i_1, \ldots, i_{n-1}, i, j \in \{0,1,2, \ldots\}$.

1. Markov Chain Assumptions

   Markov chains have several basic assumptions that you need to know, namely:

   I. The sum of state transition probabilities is 1.
   II. The probability of an event in the future does not depend on past events, but only depends on the probability of present events.
   III. The value of the transition probability from one state to another is always stationary, does not change with time.

2. Transition Probability

   The transition probability of a markov is expressed as follows:

$$P\{X_{n+1} = j | X_0 = i_0, X_1 = i_1, \ldots, X_{n-1} = i_{n-1}, X_n = i\} = P\{X_{n+1} = j | X_n = i\}$$

   It can be interpreted as follows. For a Markov chain, the conditional distribution of any future state is , X_{n+1}provided that the previous state X_0, X_1,..., $X_{n-1}$and the current state $X_n$have no influence on all previous states, and only affect the current state.

   With the above assumptions, the Markov process must have a stationary transition probability denoted by P_{ij}, then the n-stage transition probability is denoted by P_{ij} where $n = 0, 1, 2, \ldots$, n so $P_{ij}$is the transition probability of a random variable X , from state i to state j after n time

   Definition 2.1 A Markov chain $X_n$is declared homogeneous if

$$P\{X_{n+1} = j | X_n = i\} = P\{X_1 = j | X_n = i\} = P_{ij}$$

With $P_{ij} \geq 0$ $dan \sum_{j=0}^{\infty} P_{ij} = 1$for all n and all$i, j \in \{0,1,2 \ldots\}$

3. One Step Transition Opportunities

If the Markov chain is $P\{X_{n+1} = j|X_n = i\} = P_{ij}$a state space $n = 0,1,2,..\}$then the value $P_{ij}$is the probability that the state $i$will then make a transition to the state j. Let P be the transition probability matrix $m \times m$, it can be written as follows:

P =

state	0	1	…	m
0	P_{00}	P_{01}	…	P_{0m}
1	P_{01}	P_{11}	…	P_{1m}
:	:	:	:	:
m	P_{m0}	P_{m1}	…	P_{mm}

The system is in state $i$at time n, so the system will process the state $j$at time $n + 1$, , which means for every i,

$$\sum_{j=1}^{m} P\{X_{n+1} = j|X_n = i\} = 1$$

$$\sum_{j=1}^{m} P_{ij} = 1$$

4. Step Transition Opportunities

Defined for n step transition probability ${P_{ij}}^{(n)}$as the probability that the process is in the state$i$ will be in that state $j$after n additional transitions. So,

$${P_{ij}}^{(n)} = P\{X_{n+1} = j|X_n = i\} \quad i,j \in \{0,1,2 \ldots\}$$

Theorem

If ${P_{ij}}^{(n+m)} = \sum_{k=0}^{\infty} {P_{ik}}^{(n)} {P_{jk}}^{(m)}$we denote n+m transition functions and n steps of a markov, then

Proof:

$${P_{ij}}^{(n+m)} = \sum_{k=0}^{\infty} P\{X_{n+m} = j|X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_{n+m} = j, X_n = k|X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_{n+m} = j|X_n = k, X_0 = i\} P\{X_n = k|X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_{n+m} = j|X_n = k\} P\{X_n = k|X_0 = i\}$$

$$= \sum_{k=0}^{\infty} P\{X_m = j|X_0 = k\} P\{X_n = k|X_0 = i\}$$

$$\sum_{k=0}^{\infty} {P_{ik}}^{(n)} {P_{jk}}^{(m)}$$

Transition Opportunity Matrix

A Markov chain is a stochastic process { Xn , n = 0 ,1 , … } which has a state space in the form of a finite set or a numbered set. This is the result of the process of calculating on closed and finite sets of nonnegative and finite zero values (Cahyandari, 2015). For example, at time n, the process is in state k, then it can be written $Xn = k$. What is meant by a stochastic process is a collection of random variables with n representing the time index.

Dengan demikian rantai Markov dapat dituliskan sebagai berikut:
Untuk semua $k_0, k_1, \ldots, k_{n-1}, k, j$ dan semua $n \geq 0$,

$$P\left\{ \underbrace{X_{n+1} = j}_{\text{kejadian mendatang}} \middle| \underbrace{X_0 = k_0, X_1 = k_1, \ldots, X_{n-1} = k_{n-1}}_{\text{kejadian masa lampau}}, \underbrace{X_n = k}_{\text{kejadian sekarang}} \right\}$$
$$= P\{X_{n+1} = j | X_n = k\} \quad (1)$$
$$= P_{jk}$$

Based on equation (1), the conditional probability of all future events Xn +1, given past events X 0 , 1, … , Xn −1 and the current state Xn , is independent of past events, and only dependent on present events. Chance Pjk is the chance of transition to state j given the current state, which is state k. The following are properties possessed by Pj :

$$\sum_{k=1}^{m} P_{jk} = 1, P_{jk} \geq 0, j = 1, 2, \ldots, m$$

Hitting Time

There is $A \subset S$a Hitting Time $T_A$from A defined as:

$$T_A = \begin{cases} \min(n > 0, X_n \in A)\ ;\ X_n \in A, n > 0 \\ \infty\ ;\ X_n \notin A, n > 0 \end{cases}$$

with $T_A$is the first positive time the Markov chain arrives at A

Theorem

If $P^n(x, y)$the transition function has n steps of a Markov chain then:

$$P^n(x, y) = \sum_{m=1}^{n} P_x(T_y = m)\, P^{n-m}(y, y); n \geq 1$$

Proof:

Note that the events : $\{T_y = m, X_n = y\}, 1 \leq m \leq n$are mutually exclusive events.

$$\begin{aligned} \{X_n = y\} &= \{T_y = 1, X_n = y\} \cup \{T_y = 2, X_n = y\} \cup \ldots \cup \{T_y = n, X_n = y\} \\ &= \bigcup_{m=1}^{n} \{T_y = m, X_n = y\} \\ P^n(x, y) &= P_x(X_n = y) = P_x\left[\bigcup_{m=1}^{n} \{T_y = m, X_n = y\}\right] \\ &= \sum_{m=1}^{n} P_x(T_y = m, X_n = y) \\ &= \sum_{m=1}^{n} P_x(T_y = m) P(X_n = y | X_0 = x, T_y = m) \\ &= \sum_{m=1}^{n} P_x(T_y = m) P(X_n = y | X_0 = x, X_1 \neq y, X_2 \neq y, \ldots, X_{m-1} \neq y, X_m = y) \\ &= \sum_{m=1}^{n} P_x(T_y = m)\, P^{n-m}(y, y) \end{aligned}$$

The relationship between absorbing state and hitting time.

Lemma

If a is an absorbing state, then$P^n(x, a) = P_x(T_a < n), n \geq 1$

Proof:

If a state is absorbing, then$P^n(a, a) = 1; 1 \leq m \leq n$

Based on the theorem above, for y = a, we obtain:

$$
\begin{aligned}
P^n(x,a) &= \sum_{m=1}^{n} P_x(T_a = m)\, P^{n-m}(a,a) \\
&= \sum_{m=1}^{n} P_x(T_a = m) \\
&= P_x(T_a \le n)
\end{aligned}
$$

Note that:

$$
\begin{aligned}
P_x(T_y = 1) &= P_x(X_1 = y) \\
&= P(x,y) \\
P_x(T_y = 2) &= \sum_{z \ne y} P_x(X_1 = z, X_2 = y) \\
&= \sum_{z \ne y} P(x,z)P(z,y)
\end{aligned}
$$

For larger n, use the formula:

$$P_x(T_y = n+1) = \sum_{z \ne y} P(x,z) P_x(T_y = n); n \ge 1$$

The formula can be interpreted as follows:

$P_x(T_y = n+1)$ is the probability of a Markov chain starting from x reaching y in at least n+1 steps. This is the same as a chain starting from x reaching z in one step from z reaching y in n steps, as long as $z \ne y$.

RESEARCH METHODS

Data source

This research uses data on student grades from the Mathematics study program, Faculty of Mathematics and Natural Sciences, Udayana University for the courses used, namely Basic Statistics, Mathematical Statistics 1, and Mathematical Statistics 2 classes from 2015 to 2019.

Data analysis

The analysis method in this research follows the following steps:

1. Form a table of the frequency of occurrence of grades A, B+, B, C+, C, D+, D, E in Basic Statistics, Mathematical Statistics 1, and Mathematical Statistics 2 courses.
2. Determine the transition probability matrix from the table of the frequency of occurrence of values A, B+, B, C+, C, D+, D, E in Basic Statistics, Mathematical Statistics 1, and Mathematical Statistics 2 courses

RESULTS AND DISCUSSION

Formation of frequency count tables from student grade data for Basic Statistics, Mathematical Statistics 1, and Mathematical Statistics 2 courses

		SM1								
		A	B+	B	C+	C	D+	D	E	Total
SD	A	57	68	23	3	1	0	0	1	153
	B+	21	44	20	13	7	4	3	0	112
	B	4	33	12	1	3	0	1	0	54
	C+	0	1	1	0	0	0	0	0	2
	C	0	0	0	2	0	0	0	0	2
	D+	0	0	0	0	0	0	0	0	0
	D	0	0	0	0	0	0	0	0	0
	E	0	0	0	0	0	0	0	0	0
	Total	82	146	56	19	11	4	4	1	323

		SM2								
		A	B+	B	C+	C	D+	D	E	Total
SM1	A	61	20	1	0	0	0	0	0	82
	B+	30	80	23	8	4	0	1	0	146
	B	9	12	14	20	1	0	0	0	56
	C+	0	3	5	10	1	0	0	0	19
	C	0	0	3	5	1	1	1	0	11
	D+	0	0	0	3	1	0	0	0	4
	D	0	0	0	3	1	0	0	0	4
	E	1	0	0	0	0	0	0	0	1
	Total	101	115	46	49	9	1	2	0	323

Form a transition opportunity matrix

To obtain the transition probability value from the frequency count table, the formula is used $P(A) = \frac{n(A)}{n(S)}$, so that the transition probability matrix is obtained as follows:

$$SD \quad \begin{array}{c} \\ A \\ B+ \\ B \\ C+ \\ C \\ D+ \\ D \\ E \end{array} \overset{SM1}{\begin{array}{cccccccc} A & B+ & B & C+ & C & D+ & D & E \\ 0{,}373 & 0{,}444 & 0{,}150 & 0{,}020 & 0{,}007 & 0{,}000 & 0{,}000 & 0{,}007 \\ 0{,}188 & 0{,}393 & 0{,}179 & 0{,}116 & 0{,}063 & 0{,}036 & 0{,}027 & 0{,}000 \\ 0{,}074 & 0{,}611 & 0{,}222 & 0{,}019 & 0{,}056 & 0{,}000 & 0{,}019 & 0{,}000 \\ 0{,}000 & 0{,}500 & 0{,}500 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 1{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \end{array}}$$

And

$$SM1 \quad \begin{array}{c} \\ A \\ B+ \\ B \\ C+ \\ C \\ D+ \\ D \\ E \end{array} \overset{SM2}{\begin{array}{cccccccc} A & B+ & B & C+ & C & D+ & D & E \\ 0{,}744 & 0{,}244 & 0{,}012 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}205 & 0{,}548 & 0{,}158 & 0{,}055 & 0{,}027 & 0{,}000 & 0{,}007 & 0{,}000 \\ 0{,}161 & 0{,}214 & 0{,}250 & 0{,}357 & 0{,}018 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}158 & 0{,}263 & 0{,}526 & 0{,}053 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}273 & 0{,}455 & 0{,}091 & 0{,}091 & 0{,}091 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 0{,}750 & 0{,}250 & 0{,}000 & 0{,}000 & 0{,}000 \\ 0{,}000 & 0{,}000 & 0{,}000 & 0{,}750 & 0{,}250 & 0{,}000 & 0{,}000 & 0{,}000 \\ 1{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 & 0{,}000 \end{array}}$$

Based on this matrix, the probability that a student who gets an A in the Basic Statistics course will get an A grade in the Mathematical Statistics I course is 0.373, and has a chance of 0.444 for a B+, 0.150 for a B, 0.020 for a C+, 0.007 for a C and 0.007 for E. The chance of a student getting an A in Mathematical Statistics I and again getting an A in Mathematical Statistics II is 0.744, and has a chance of 0.244 and 0.012 to get a B+ and B.

Then, if a student gets a B+ grade in the Basic Statistics course, the chance of the student getting an A grade in the Mathematical Statistics I course is 0.188 , the chance of getting the same grade is 0.393, the chance of getting a B grade is 0.179, a C+ grade is 0.116, a C grade of 0.063, the D+ value is 0.036 and the D value is 0.027. If a student gets a B+ in the Mathematical Statistics I course then the chances of getting A, B+, B, C+, C, and D in the Mathematical Statistics II course are 0.205, 0.548, 0.158, 0.055, 0.027 and 0.007

If a student gets a B grade in the Basic Statistics course, the chances of the student getting A, B+, B, C+, C, and D grades in the Mathematical Statistics I course are 0.074, 0.611, 0.222, 0.019, 0.056, and 0.019. If a student gets a B in the Mathematical Statistics I course, the chances of getting an A, B+, B, C+, and C in the Mathematical Statistics II course are 0.161, 0.214, 0.250, 0.357, and 0.018.

CLOSING

From the research conducted, it can be concluded that the chance of a student getting an A grade when they get a grade below B is very small or even non-existent. However, when you get an A, the chances of getting a grade other than A are quite large. So that the score obtained from Basic Statistics can influence the score obtained in Mathematics Statistics II.

BIBLIOGRAPHY

Cahyandari, R. (2015). Study of point process formation using a counting size approach. Journal of Islamic and Technology Studies (ISTEK), Faculty of Science and Technology, State Islamic University.

Lefebvre, M., & Kounta, M. (2011). First hitting problems for Markov chain that converge to a geometric Brownian motion. International Scholarly Research Network ISRN Discrete Mathematics.